# Application of system dynamics to assess mass flows of waste electrical and electronic equipment (WEEE)

Sinha Khetrival, Deepali ; Widmer, Rolf ; Schwaninger, Markus ; Hilty, Lorenz

Abstract: The consumption and disposal of electrical and electronic equipment forms a dynamic system. This paper applies system dynamics methodology to assess mass flows of end-of-life equipment. In the paper, two modelling approaches to forecasting disposal of consumer durables are discussed, namely the "delay model" approach and the "reverse diffusion model" approach. Applying the same dataset on the disposal of cathode ray tube personal computer monitors in Switzerland to both the approaches, the estimates and forecasts of the models are compared against real system data. The comparison provides an opportunity to discuss further improvements to both modelling approaches.



Originally published at:
Sinha Khetrival, Deepali; Widmer, Rolf; Schwaninger, Markus; Hilty, Lorenz (2012). Application of system dynamics to assess mass flows of waste electrical and electronic equipment (WEEE). In: The 30th International Conference of the System Dynamics Society, St. Gallen, 22 July 2012 - 26 July 2012.

# Application of System Dynamics to Assess Mass Flows of Waste Electrical and Electronic Equipment (WEEE)

Deepali Sinha-Khetrival, Rolf Widmer, Markus Schwaninger, Lorenz M. Hilty

## Abstract

The consumption and disposal of electrical and electronic equipment forms a dynamic system. This paper applies system dynamics methodology to assess mass flows of end-of-life equipment. In the paper, two modelling approaches to forecasting disposal of consumer durables are discussed, namely the “delay model” approach and the “reverse diffusion model” approach. Applying the same dataset on the disposal of cathode ray tube personal computer monitors in Switzerland to both the approaches, the estimates and forecasts of the models are compared against real system data. The comparison provides an opportunity to discuss further improvements to both modelling approaches.

## Introduction

Consumer durable goods – from household appliances such as refrigerators and vacuum cleaners to entertainment electronics such as televisions and music players to information technology products such as computers and printers – have proliferated over the last fifty years. Not only have the number and types of products multiplied, their increased affordability has seen sales of consumer durables skyrocket. Estimates by Euromonitor (2011) suggest that global sales of consumer electronics were to the tune of 3 billion units in 2010.

The flip side to increasing consumption of durable goods is the growing volume of electronic waste, or e-waste. E-waste has gained importance over the past decades because not only does it contain toxins which can be harmful to human health and the environment, but it also contains valuable precious metals and rare earths which are critical for the manufacture of new generation consumer durables. For example, by some estimates, the global material supply of critical metals in the manufacture of electronics such as Indium might be exhausted before the end of the decade (UNEP, 2009). The reservoirs of e-waste are therefore increasingly seen as valuable “mines” to recover scarce elements from.

Over the past decade, these social, environmental and economic drivers have necessitated better management of e-waste, spurring legislation such as the European Union’s Waste Electrical and Electronic Equipment (WEEE) Directive, the Japanese Specified Home Appliances Recycling Law (SHAR) and the Swiss Ordinance on the Return, Taking back and Disposal of Electrical and Electronic Equipment (ORDEE), among others, which are specifically aimed at end-of-life consumer durables.

Essential for sustainable and efficient management of e-waste is an accurate estimation of the timing and quantity of e-waste disposals (Kang and Schoenung, 2006). Models, as simplified representations of real systems, can both reproduce or recreate (“portrait”) and anticipate (“paragon”) and are crucial in providing an understanding of the real system (Schwaninger, 2010). To support decision making efforts for policy makers (for example in setting targets), recyclers (for estimating availability of supply) or take-back systems (to estimate logistical and cost implications, several authors such as Widmer et al. (2005) Oguchi, et al. (2008) Yang, et al. (2008) have previously presented models forecasting e-waste flows.

A system dynamics approach is well suited to assess mass flows linked to e-waste, given there are quite accurate data on sales figures, market penetration and life time of electrical and electronic equipment (EEE). Essentially, the system is sufficiently described with three variables as follows:

- ***Stocks:*** Stocks are the installed base of a product in households
- ***Inflows:*** Inflows are sales or shipments of new consumer durable products
- ***Outflows:*** E-waste disposals of end-of-life consumer durables

The stock-flow diagram below graphically illustrates the relationship between inflows, stocks, outflows.

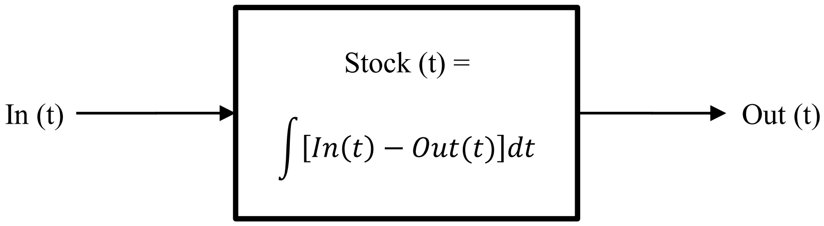


***Figure 1:*** Generic stock-flow model where the outflow is moderated by the inflow and/or the stock and/or a disposal probability density

It is possible to consider various scenarios to estimate and forecast mass flows of e-waste, depending on the data available. In the best case, sufficient time series data on all four variables is available, making it possible to not only estimate, but also validate the model forecasts. However, it is more common to have insufficient time series data for one or more variables. We discuss two models to estimate the mass flows of e-waste in two scenarios of partial data availability. In the first scenario, stock and inflow and perhaps failure or obsolescence rate data is available, and in the second scenario, time series data on outflows is available and perhaps data on stock. To assess e-waste mass flows in the two scenarios, we discuss two models, namely the "delay model" and the "reverse diffusion model".

| | **Stock** | **Inflow** | **Outflow** | **Failure or Obsolescence Rate** | **Model to assess e-waste mass flows** |
|---|---|---|---|---|---|
| Scenario 1 | Known | Known | Unknown | Known/ Unknown | Linear time invariant (LTI) system (delay model) |
| Scenario 2 | Known/ Unknown | Unknown | Known | Unknown | Ricatti equation system (reverse diffusion model) |

***Table 1:*** Data availability scenarios

The "delay model", also sometimes referred to as the market supply model (Widmer et al., 2005), is the most commonly used modelling approach to estimate disposals of consumer durable product. Delay models assume the system is linear and time invariant (LTI). An LTI system is called causal if the output value at any time *t* depends only on input values for times less than $t$. Thus, in the case of delay models, e-waste flows at any time $t$ are dependent only on sales of electronic products in the past and delay time.

The second modelling approach discussed and compared in this paper is the “reverse diffusion” approach which suggests that the cumulative disposal of old consumer durables tends to follow an sigmoidal curve which is exponential at first and then gradually slows until the entire stock is exhausted, that is maximum carrying capacity is reached.

The goal of the paper is to review and compare the two modelling approaches used to estimate and forecast e-waste mass flows. Data on the sales and disposal of Cathode Ray Tube monitors (CRT monitors) for personal computers (PCs) in Switzerland is applied to both model approaches to examine their differences and compare the performance of outputs of the models against a real system.

## A Linear Time Invariant (LTI) system (delay model):

When stocks and inflows are known, the delay model can be used to assess mass flows of e-waste. The relationship between stocks and flows in the delay model are represented by:

In differential form:

$$dS(t) = In(t)dt - Out(t)dt \tag{1}$$

In integral form:

$$S(t) = \int_{t_0}^{t} \big(In(\tau) - Out(\tau)\big)d\tau \tag{2}$$

As a recurrence relation (to numerically integrate the differential equation (1)):

$$S_{t+1} = S_t + (In_t - \ Out_t)\Delta t \tag{3}$$

where $S(t), S_t$ represent the stock, $In(t), In(\tau), In_t$ represent the inflow and $Out(t), Out(\tau), Out_t$ represent the outflow.

Outflows (disposals) of consumer durables are a function of inflow (sales) and of the residence time (given as a survival or reliability function *R(t)*) of the product in a household. As this model assumes that all products are disposed of at end of life the lifetime distribution function

$$F(t) = 1 - R(t)$$

is also the disposal distribution *D(t)* and its derivative, the disposal density function *d(t)*, describes the rate of disposals.

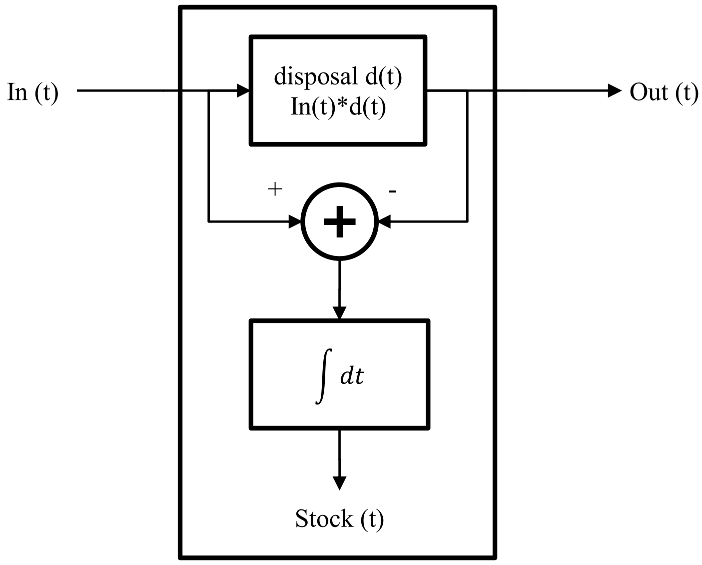


***Figure 2:*** Stock and flow diagram for the delay model ('*' represents a convolution)

The outflows (disposals) $Out$ are then expressed as a convolution of the inflows (sales) *In* and the disposal density *d*,

In discrete form the convolution writes:

$$(f * g)[k] = \sum_{i=-\infty}^{\infty} f[i] \cdot g[k-i] \tag{4}$$

Thus,

$$Out_t = \sum_{i=-\infty}^{\infty} d_i \cdot In_{t-i} \tag{5}$$

where $Out_t$ is the number of products disposed and $d_t(i)$ the disposal function.

If both $In_t$ and $Out_t$ are zero outside a time window $T_{min} < i, t < T_{max}$ then the functions are causal and the integration interval can be limited to $[2T_{min}, 2T_{max}]$.

The outflow in the delay model is solely determined by the inflow and the residence time distribution of products in households. The latter is defined as the probability that the time of obsolescence is later than some specified time *t*.

$$F(t) = D(t) = 1 - R(t) \tag{6}$$

and (7)

$$D(t) = \int_0^t d(\tau)d\tau$$

in discretized form:

$$D_t = \sum_{i=0}^{t} d_i \qquad (8)$$

where $d_i$ is the discrete disposal density of the consumer durable.

The rate of disposal of a consumer durable device can be described by many probability densities in an analogous way to product failure. This is a reasonable assumption given that disposals are either due to technical failure or discretionary obsolescence, both of which are correlated with the product's age (Steffens, 2003). Following its acceptance in failure analysis, the Weibull distribution is the most commonly used distribution to model the lifetime distribution (Oguchi et al., 2008; Walk, 2009; Yu et al., 2010).

Therefore, in this model, a Weibull distribution is used for the disposal distribution function; it's discrete density is given by the equation:

$$d_i = \frac{\gamma}{\alpha}\left(\frac{i}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{i}{\alpha}\right)^{\gamma}} \qquad (9)$$

where $\gamma$ is the shape parameter and $\alpha$ the scale parameter of the disposal function.

## A Ricatti Equation System (reverse diffusion model):

The diffusion of consumer durables has been extensively modelled, in particular by the Bass diffusion model (Bass, 1969) which has been empirically validated across a range of electrical and electronic products. The reverse diffusion model builds on the extant literature on demand forecasting of new products based on well documented diffusion models on the basis that the dynamics of disposal of consumer durables are not dissimilar to the diffusion of new products.

In the reverse diffusion model, the market depletion of a durable is considered to be driven by two main reasons, characterised by the parameters p and q:

1. ***Technical disposals***: characterised by *p*, considered the co-efficient of technical disposal, which is the influence of factors such as breakdown and wear and tear on e-waste flows
2. ***Discretionary disposals***: characterised by *q*, considered the co-efficient of discretionary disposal, and influenced by social and functional signals such as status, new product technologies, etc.

The underlying structure of the reverse diffusion models is illustrated in the diagram below.

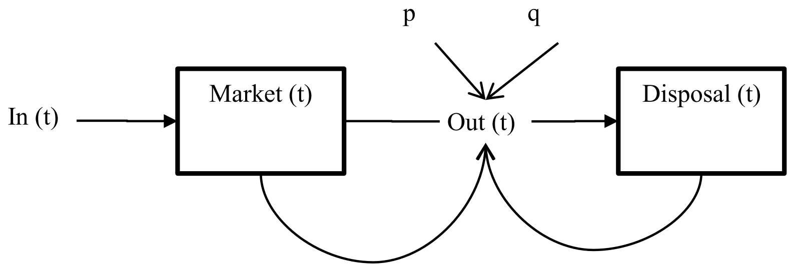


***Figure 3***: Reverse Diffusion Model Structure

The reverse diffusion differential equation is given as:

$$\dot{S} = p \cdot (1 - S) + qS \cdot (1 - S) \tag{10}$$

and has the following solutions.

$$D(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \tag{11}$$

$$d(t) = \frac{(p+q)^2}{p} \frac{e^{-(p+q)t}}{\left(1 + \frac{q}{p} e^{-(p+q)t}\right)^2} \tag{12}$$

where $D(t)$ is the cumulative disposal function, $d(t)$ is the disposal density function, i.e. the specific disposal rate.

The absolute disposal rate, $\hat{d}(t)$ is given by:

$$\hat{d}(t) = m \cdot d(t) \tag{13}$$

where m is the total number of the durable products in the market stock which is to be depleted to the disposal stock.

## Experimental Frame

The models discussed above are compared to each other by applying them to the case of the disposal of Cathode Ray Tube (CRT) monitors in Switzerland. CRTs had been the dominant display technology for both televisions and Personal Computer (PC) monitors until they were rendered obsolete with the advent of new flat panel display technologies.

Personal computers were introduced to the Swiss market in the early 1980s. Disposals of these products followed soon after, and by 1994, there was a formal collection and take-back system to ensure sound disposal of end-of-life PCs. With reliable data on sales, stocks and specially disposals, estimating and forecasting flows of CRT monitors makes an excellent case study to compare and validate the models discussed above.

Data on the stock and inflow (sales) of PC monitors was obtained from Robert Weiss Consulting (www.weissbuch.ch), while data on outflows (disposals) was acquired from SWICO Recycling, the Swiss producer responsibility organisation that manages the collection and take-back system for several consumer durables, including IT equipment such as PC monitors and EMPA, who provide the monitoring and control for the SWICO system.

## Results

Figure 4 and Figure 5 show the results in applying the two modelling approaches to CRT monitor glass stocks and flows through Switzerland. In the delay model, the inflows are convolved with the disposal probability distribution, a Weibull distribution which parameters are optimised with a least square fit to match the actual stock and the sales. In the reverse diffusion model again the parameters of the Bass equation are optimized with a least square fit to match the actual disposal flow.

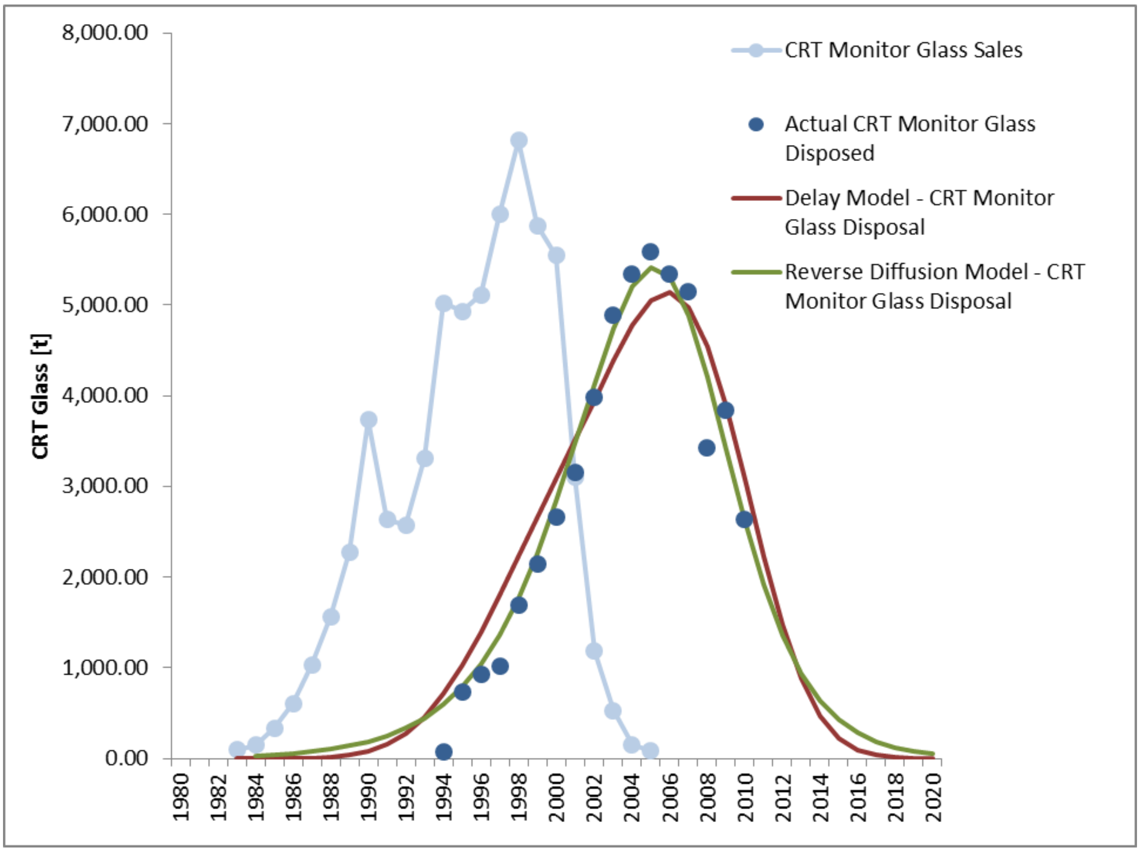


***Figure 4:*** Actual CRT monitor glass masses sold to and disposed from the market in Switzerland and the disposal estimates of the delay model (red) and the reverse diffusion model (green)

Parameter and fit statistics for both models is given in the table below. α and γ are the scale and shape parameters of the Weibull distribution respectively, and p and q are parameters of the reverse diffusion model. α is interpreted as the average lifetime of the product, with γ the deviation from average lifetime. Both model estimates show a good fit to actual data, with $R^2$ above .90.

| | $\alpha$ (scale parameter) | $\gamma$ (shape parameter) | $R^2$ |
|---|---|---|---|
| **Delay Model** | 9.72 | 3.88 | 0.91 |
| | ***p*** **(co-efficient of technical disposal)** | ***q*** **(co-efficient of discretionary disposal)** | $R^2$ |
| **Reverse Diffusion Model** | 0.0005 | 0.35 | 0.96 |

***Table 2:*** Parameter values and fit statistics

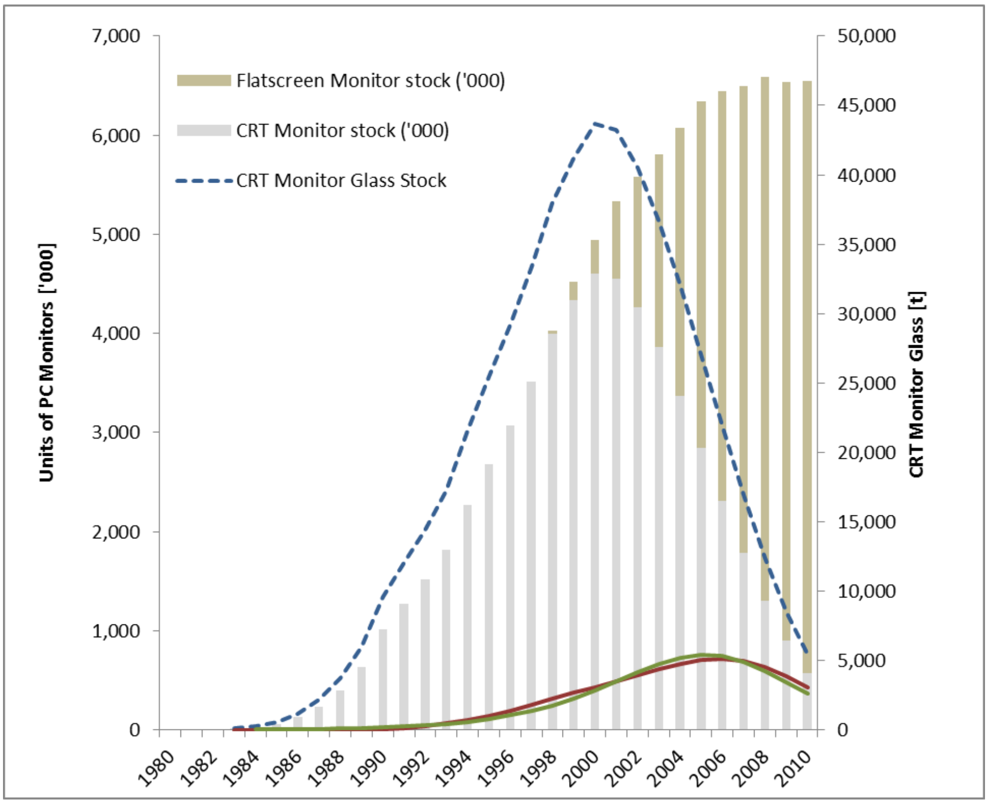


***Figure 5:*** Actual CRT monitor glass masses stored in the market in Switzerland (dotted blue line) and the prediction of the delay model (red) and the reverse diffusion model (green).

## Discussion

Comparing ***Figure 2*** and ***Figure 3*** it can be seen that both models are structurally very different. In the delay model, e-waste mass flows are expressed as a convolution of the inflows *In* and the disposal density *d*, and entirely independent of the stock. In comparison, in the case of the reverse diffusion model, the outflow is independent of the inflow. The outflow feeds a disposal stock which determines, together with the parameters *p* and *q* the dynamics of the outflow. The total stock in the market which is to be depleted (carrying capacity) to the disposal stock scales the outflow.

Both models presented above are simple and parsimonious and can be particularly useful for established take-back and collection systems with data over several years. The parameters can be updated every year by fitting to the latest available data, thereby enabling ever more accurate forecasts for the years ahead.

However, neither model shed any light on why and how consumers dispose of their durable goods, especially when they are still functional. Neither model provides any insight into the factors that influence disposal decisions or the alternatives to disposal that consumers may consider.

## Limitations/ Further Research

Both models discussed above are not without limitations. These limitations, however, could provide directions for future research.

- Both models do not incorporate the time-varying nature of consumer behaviour. Anecdotal evidence for PCs and mobile phones has shown that average lifetime of these products has reduced over time, with more frequent replacements and disposals taking place.
- Both models do not distinguish between in-use and storage time during which consumers keep old products idle before disposing them of and use a new one.
- It is likely that the collection efficiency of the system biases the disposal results, as collection data would not reflect the true disposals especially in the case of any leakages from the system. End-of-life consumer durables such as TVs and PCs are often shipped from developed countries to developing countries illegally, thereby are not accounted for in the formal collection system. A model fitted to only collection data from legal, formal systems risks underestimating actual disposals.

Further research into consumer behaviour to get insights into why, when and how consumers dispose their durable products, will provide useful information that could lead not only to better, more accurate forecasting models, but also inform consumer education and awareness programs directed towards improving consumer attitudes towards disposal of durable products.

## References

Bass, Frank M. 1969. ‘A New Product Growth for Model Consumer Durables’. *Management Science* 15 (5, Theory Series) (January): 215–227.

Euromonitor. 2011. ‘Consumer Electronics 2010’.

Kang, H. Y, and J. M Schoenung. 2006. ‘Estimation of Future Outflows and Infrastructure Needed to Recycle Personal Computer Systems in California’. *Journal of Hazardous Materials* 137 (2): 1165–1174.

Oguchi, Masahiro, Takashi Kameya, Suguru Yagi, and Kohei Urano. 2008. ‘Product Flow Analysis of Various Consumer Durables in Japan’. *Resources, Conservation and Recycling* 52 (3) (January): 463–480. doi:10.1016/j.resconrec.2007.06.001.

Schwaninger, M. 2010. ‘Model-based Management (MBM): a Vital Prerequisite for Organizational Viability’. *Kybernetes* 39 (9/10): 1419–1428.

Steffens, P. R. 2003. “A model of multiple-unit ownership as a diffusion process.” *Technological Forecasting and Social Change* 70 (9): 901–917.

UNEP. 2009. ‘Recycling – From E-waste to Resources’.

Walk, W. 2009. “Forecasting quantities of disused household CRT appliances–A regional case study approach and its application to Baden-W\ürttemberg.” *Waste Management* 29 (2): 945–951.

Widmer, R., H. Oswald-Krapf, D. Sinha-Khetriwal, M. Schnellmann, and H. Böni. 2005. ‘Global Perspectives on E-waste’. *Environmental Impact Assessment Review* 25 (5): 436–458.

Yang, J., B. Lu, and C. Xu. 2008. ‘WEEE Flow and Mitigating Measures in China’. *Waste Management* 28 (9): 1589–1597.

Yu, J., E. Williams, M. Ju, and Y. Yang. 2010. ‘Forecasting Global Generation of Obsolete Personal Computers’. *Environmental Science & Technology* 44 (9): 3232–3237.